MATHEMATICS SYLLABUS D

Paper 3171/22
Paper 22

Key messages

Candidates need to appreciate the importance of using values to more than 3 significant figures, or more than 1 decimal place for angles, for intermediatory steps. This was particularly evident on **Question 2(c)**; candidates working with 3 significant figures found a final answer correct to 1 significant figure only. In questions where candidates are asked to show that an answer is a particular value to a given degree of accuracy, it is important for candidates to determine the value to more accuracy than is required in order to show that this value rounds to the given value.

General comments

Overall, the presentation of candidate's work was good, however some candidates chose to overwrite a first attempt and so make work difficult to mark and decipher the answer the candidate intended. Occasionally the working was difficult to follow as the steps were scattered throughout the working space. This was particularly noticeable on **Questions 2(a)** and **6(a)**. Candidates should be encouraged to use a sharp pencil when drawing diagrams, particularly graphs. Use of large dots when plotting points on graphs can mean that when a tangent is drawn there may be an airgap between their tangent and their curve.

Comments on specific questions

Question 1

(a) (i) Most candidates were able to plot all six points correctly, within tolerance, with very few mis-plots seen. Some candidates did not attempt to plot any points.

(ii) Most candidates identified the correlation correctly as positive correlation. A minority of candidates wrote that the correlation was negative or, less commonly, that there was no correlation. Some answered with words such as increasing or upward.

(iii) Most candidates drew a line of best fit within the tolerance allowed. Most candidates attempted a ruled line with a positive gradient, however forced the line through the origin. Very few attempts were seen of lines with a negative gradient, but more chose to join their plots with multiple line segments, rather than a single straight line.

(iv) The majority of the candidates could accurately read their graph at d = 30.

(b) (i) Most candidates gave the correct probability. The most common error was the premature approximation to 36.6. There were some candidates who performed incorrect calculations such as: $\frac{42+36+19}{150}[\times 100]$; $\frac{36+19}{100}[\times 100]$ or $\frac{36}{150}[\times 100]$.

(ii) Most candidates applied the correct method to calculate an estimate for the mean. Some candidates obtained the sum of the correct midpoints and divided by 5. Others calculated some or all of the midpoints incorrectly, obtaining values outside the limits of some or all of the ranges, such as 10, 10, 5, 5, 30 or 5, 15, 5, 5, 45. A few candidates used the limits of the ranges as their midpoints in an otherwise correct method. Some candidates did not use the 150 given in the question but added the frequencies together incorrectly and so their calculation involved a division by an incorrect value.

Question 2

(a) Many candidates did not correctly calculate the number of hours worked each day, with errors seen in finding the number of hours for the afternoon's work and also in adding the two time periods together. Converting a time to hours and minutes was challenging for some; times involving 15 minutes as 0.15 hours and 30 + 45 = 75 minutes = 0.75 hours were very common. Some candidates attempted to convert the time to minutes and calculate the pay rate but did not work to sufficient accuracy to achieve the correct solution. There were several candidates who gave their answer to 3 significant figures rather than as an exact value.

(b) Most candidates correctly calculated the amount of pay left after paying tax. Common errors usually subtracting 7.5 from the gross pay of 544 and in some cases multiplying the gross pay by 5 or 7, even though the working week was given as 40 hours.

(c) This part of the question was more demanding. Most candidates made a successful start by calculating the balance for the simple interest investment. Common errors at this stage usually involved stopping at the interest only and in a small number of cases finding the interest for just one year. Those who knew the correct formula for compound interest correctly equated this with their previous answer. Solving the resulting equation proved a challenge to many. Simplifying $\left(1+\frac{p}{100}\right)^5$ as $1^5+\frac{p^5}{100^5}$ was often seen. Many of those who correctly obtained $\sqrt[5]{1.105}$ often rounded the value to 1.02 and were then unable to give a final answer to the required accuracy.

Question 3

(a) The majority of candidates obtained the correct y value. The common incorrect answers seen were –5.5 and –3.5.

(b) The curves drawn by the candidates were generally accurate and of a good standard. Some candidates did not obtain full marks as their curve either went out of the domain or had a considerable amount of feathering. The more common mistakes with plotting were the mis-plotting of y = –1.8 or 3.8 and plotting at (–3, –5.5) or (0, 0)

(c) The majority of candidates made good attempts at drawing the tangent. Some candidates had an air gap between the curve and the tangent which was more common when candidates had plotted their points as large closed circles and drawn the tangent at the edge of these circles rather than using the curve. Errors in the value of the tangent were usually the result of inaccurately drawing the curve, resulting in a value outside of the acceptable range. It was not uncommon to see answers with a positive gradient or a negative gradient in the working and a positive gradient on the answer line.

(d) Few candidates obtained fully correct solutions to this part. Many candidates drew a straight line somewhere on the graph, many drew horizontal lines (commonly x = 2 or x = 1). There was also a good number who drew curves instead of a suitable line. Candidates appreciated the need to read values from the graph at the intersection of their line and curve but several made the assumption that they needed to read the values where the curve crossed the x-axis. Some candidates tried to calculate the solutions rather than using their graph, mostly unsuccessfully.

Question 4

(a) (i) Most candidates had a good understanding of Venn diagrams with the diagrams either being correct or with at most two errors. It was common to see $(P \cup F)'$ left blank. There were a number of candidates who made multiple errors with the numbers being written in several places on the diagram.

(ii) The majority of candidates were able to identify the correct subset and list the correct values or follow through from their incorrect diagram.

(iii) This question proved challenging for many candidates. Some candidates were able to identify the correct subset but listed its elements. Two of the most common errors involved answers of 3 and the integers 4, 8, 10. These seem to imply that candidates misinterpreted $P' \cup F'$ as $(P \cup F)'$ and in some cases the n(….) notation by writing the elements rather than the number of elements.

(b) (i) Most candidates gave the correct answer of $\frac{3}{10}$. Many gave the answer as $\frac{2}{10}$, presumably not realising that 1 is a square number.

(ii) Only a third of the candidates coped well with the two events and obtained the correct simplified fraction. Some candidates gave the correct method of $\frac{2}{10} \times \frac{2}{10}$ but then evaluated this as $\frac{4}{20}$. Other common errors included $\frac{2}{10} + \frac{2}{10}$, $\frac{2}{10} \times \frac{1}{10}$ and giving a final answer of $\frac{1}{45}$ leading from not replacing the first card.

(iii) Relatively few candidates obtained the correct probability, although a number of partially correct solutions were seen. Most candidates with a correct answer opted to use the method P(1 or 2 cards greater than 7) with the method 1 – P(no cards greater than 7) only seen on a few occasions. Examples of partially correct solutions were $\frac{3}{10} \times \frac{2}{9}$ alone, $\frac{3}{10} \times \frac{2}{9} + \frac{3}{10} \times \frac{7}{9}$, $\frac{3}{10} \times \frac{7}{9}$ alone or $2\left(\frac{3}{10} \times \frac{7}{9}\right)$. A few candidates treated this as a question involving replacement and obtained the answer $\frac{51}{100}$. Some candidates added probabilities rather than multiplying and others chose to multiply three probabilities, for example $\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}$.

Question 5

(a) Few candidates correctly demonstrated how to obtain the value of *d* to at least 4 significant figures and round this value to 3 significant figures. The most common error was to try and find the area of the trapezium cross-section to then to find *d*, with many attempts using 0.477 to show that the value of *d* is 0.477. Of those who appreciated the need to use Pythagoras' theorem, the most common error was using 0.3 in their expression rather than 0.15. Many of those who understood how to calculate *d* failed to show a value to more than 3 significant figures.

(b) Many candidates were able to find the surface area of the tank. The formula for the area of a trapezium was well known but several used the height as 0.5 instead of 0.477. Not all candidates appreciated that *BCGF* was a rectangle and treated it as a trapezium. Other mistakes included missing one of the rectangles of size 0.5 m by 1.6 m or including the rectangle of size 0.95 m by 1.6 m.

(c) (i) The most successful solutions used the similar triangle approach but a common error was to think the trapezia were similar. Some chose to look at the area of the large trapezium but not all equated this to the sum of the two smaller trapezia, often only equating it to one of these. Those who set up the equation correctly often made errors and obtained a value out of range.

(ii) Few candidates realised that the surface of the water was a rectangle and used the correct values to obtain its area. Many used the trapezium rule again with sides of 0.65 m and 0.84 m to give an area of 0.2235 m^2.

Question 6

(a) Most candidates made some attempt to find the time required, with about half of them successfully finding this quantity. The majority of the candidates understood that to find the time they needed to divide the distance by the speed. Some candidates did not correctly convert the units from kilometres to metres and from hours to seconds. Often the interim calculations lacked the necessary accuracy to obtain a time correct to the nearest second. Some candidates who obtained a time in minutes assumed that the decimal part of that time was the time in seconds, for example 1.47 minutes is 1 minute 47 seconds.

(b) (i) Trigonometric ratios were used by most candidates for this part, although the sine rule was occasionally seen. Common wrong errors were $\tan\theta = \frac{AD}{285}$, sometimes from the wrong labelling of the angle 35°, or $\sin\theta = \frac{AD}{285}$. A number of successful solutions calculated *BD* initially and then used either Pythagoras' theorem or the cosine ratio to find *AD*. There were several candidates who obtained an answer out of range due to the use of premature approximation, usually for $\tan\theta$.

(ii) Nearly half the candidates showed excellent skills with this complex contextual question that needed breaking down into several parts. Most candidates chose to use *AD* and Pythagoras' theorem to calculate *BD*. Most also realised that they needed to add 35 to their value for angle *BDC*. Attempts to find angle *BDC* were not always correct; errors seen included an incorrect formula for the cosine rule, or incorrect rearrangement of the cosine rule, or assuming that triangle *BDC* was a right-angled triangle. Some candidates correctly calculated angle *BDC*, however they stated that angle as the required bearing. Inaccuracies or approximations in the calculations sometimes led to a bearing out of the required range.

Question 7

(a) (i) Many candidates gave the correct expression for the nth term. Some candidates reached the expression $1 + (n - 1)6$ but in some cases the resulting expansion and simplification was carried out incorrectly. Common errors included $n + 6$ and answers that involved finding extra numerical terms.

(ii) Success in this part was dependent on a correct expression for the nth term in the previous part and only a minority of those candidates were able to give a fully correct explanation. Some common errors included: substituting some values of n to obtain terms that were odd numbers; using the sequence with a comment that 6 was being added each time; not mentioning that 5 is odd or $6n$ is even.

(b) Few candidates were able to give fully correct expressions for the sequence. Most candidates made progress in recognising that the denominator was a quadratic and, in some cases, identifying $(n + 2)^2$ as the correct expression. The numerator proved more challenging, with very few recognising it as a geometric sequence. The most common incorrect expression for the numerator was $2n$.

(c) This part of the question proved challenging. Some candidates were able to set up the correct quadratic and solve it correctly. Not all of these candidates realised that n was an integer and answers of 26.8 were often seen. The quadratic was not always set to 2000, with some setting it to 2001 before solving the resulting equation while others set the equation to 0. A few candidates chose to use trial and improvement to obtain the solution.

Question 8

(a) (i) The majority of candidates used the formula for the distance between two points correctly to obtain $\sqrt{29}$, however, many of them did not give an accurate value to at least 3 significant figures. Many errors were seen due to a lack of accurate brackets when dealing with negative numbers.

Misunderstandings were seen in calculating the distance, for example, using $\sqrt{x+y}$ or $\sqrt{x^2-y^2}$ or using a calculation involving squaring each of the coordinates.

(ii) This part was attempted by most candidates, but few drew a diagram to help visualise the problem or gave relevant working that led to the correct answer. The majority wrongly tried to use Pythagoras' theorem to calculate the coordinates of *C*. The correct use of vectors and therefore the correct answer were rarely seen.

(b) (i) Most candidates understood how to find the midpoint of a line, although occasional errors of (6, 1) were seen. Errors in the handling of negative numbers were sometimes seen.

(ii) The vast majority of those candidates who attempted this question correctly found the gradient of *PQ*. Some used this to find the equation of *PQ*, which was unnecessary. A large number did not continue on to find the gradient of the perpendicular and so did not obtain the required equation. Those candidates who did continue usually knew the formula relating the gradients and used this correctly. Having obtained a gradient for the perpendicular, most knew how to then find an equation but several used either *P* or *Q* rather than the midpoint.

Question 9

(a) Few candidates produced a fully correct proof of these congruent triangles. The most common error was the use of incorrect or insufficient reasons to justify statements and not giving, or giving the incorrect, congruency conditions. Generally, candidates were able to justify only one equal pair sufficiently or sometimes state two equal pairs. Many candidates could list two or three correct pairs, or could state that *OA* and *OB* were radii, but fewer recognised that *OA* = *OB* and *AX* = *BY* with correct reasons. Valid reasons for *OX* = *OY* and angle *AXO* = angle *BOY* were few. Even when three equal pairs had been fully justified, proofs often lacked a conclusion or featured a mismatched conclusion with the chosen justified pairs.

(b) (i) Whilst most candidates obtained the correct answer, errors included: finding $\tan^{-1}\left(\frac{3.5}{4}\right)$ or calculating $\cos^{-1}\left(\frac{3.5}{4}\right)$ without further working; not doubling a correctly found angle *AOX*; incorrectly rearranging the implicit form of the cosine rule or using the cosine rule to find the wrong angle without further working. Premature approximation also led to incorrect answers.

(ii) Candidates who attempted this question usually made some progress towards finding the shaded area. Errors included: after finding one of the shaded areas correctly, not doubling to find the whole area; a mismatch in areas used in their subtraction, for example doubling the area of one sector but not the area of *AOB* and vice versa; only finding one of the necessary areas; using 4 as the height of the triangle giving rise to $\frac{1}{2}\times 4\times 7$ or $\frac{1}{2}\times 4\times 3.5$ being seen as the areas of the triangles or neglecting to find one of the component areas for this method.

Question 10

(a) In general, this question was answered well with many candidates giving a fully correct equation with its solution. Common errors occurred through not reading the question properly with an expression for Farah's starting number being given as $(4 - n)11$ or $n - 4 \times 11$ and Abid's as $5(n + 1)$. Some candidates chose to put each expression equal to y but then few could make progress from that point.

(b) This part was answered well by the majority of the candidates. The most common errors seen were in the expansion of $a(c + 1)$ to $ac + 1$ or $b(4 - c)$ to either $4b - 4c$ or $4b - c$. The collection of terms was sometimes spoilt by candidates failing to change the signs when moving terms to the opposite side of the equation. Those who had collected terms were usually able to factorise and get an equation with c as the subject. Having done the expansion, some candidates left a term in c on each side of the equation so their final expression for c was a function of c.

(c) Many candidates were able to answer this part well. There were some who were able to obtain a correct numerator and denominator and then either made a mistake simplifying or having simplified correctly then went on to produce further work which resulted in an incorrect fraction. The most common error was to see –8 in the numerator instead of +8. Candidates who left the denominator as the product of two brackets were more successful than those who attempted to expand the brackets as many introduced errors as a result.